ACTUAL GAS CONTENT OF VERTICAL ADIABATIC TWO-PHASE FLOWS AT LOW PRESSURE AND IN VACUUM

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Generalized equations are given for determining the actual gas content of low-viscosity and high-viscosity two-phase flows in vertical tubes and annular channels with D_{equ} = 6-1000 mm.

The experimental data on the motion of low-viscosity and high-viscosity two-phase flows at low pressure and in vacuum in tubes and in annular channels with $6 \leqslant D_{equ} \leqslant 1000$ mm (see, e.g., [1-8]) accumulated over the past decade have not yet been adequately analyzed and generalized. Our purpose here is to bridge this gap.

The governing properties adopted for studies of the relative motion of the phases in vapor—liquid (or gas—liquid) flows are the actual gas content φ, the relative velocity of the different phases, w_r (w_r/w_m in dimensionless form), the actual gas velocity $w_2(w_2/w_m)$, and the phase "slipping" $s = w_2/w_1$.

In general, each of these characteristics depends on many parameters [9, 10],

$$\pi = f(\beta,\ Re_m, Fr_m,\ We,\ \bar{\rho},\ \bar{\mu},\ k/D,\ \alpha), \tag{1}$$

and this circumstance naturally complicates the design of experiments and the analysis of experimental results.

In the case under consideration here, there are some specific difficulties, resulting from the following circumstances: It is not possible to arbitrarily change Fr_m so that Re_m and We remain constant. Variability of Fr_m and Re_m at constant We can be achieved by means of w_m, but then we would be dealing with the influence on the process of the dimensional parameter w_m, rather than the influence of Re_m and Fr_m at We = const. We therefore use dependence (1), after converting it to a form more convenient for generalizing the concrete experimental results.

The experimental characteristic is shown in Table 1, where the values of P without asterisks denote the average pressure in the experimental region, while the values of P with asterisks denote the pressure in the separator. The surface tension at the interface varied slightly: $\sigma = (7.11\text{-}7.74)\cdot 10^{-2}$ N/m.

Since the experimental data used in the generalization have already been analyzed thoroughly [1-8], here we will simply discuss certain particular features of the influence of the physical properties of the flow and the geometric dimensions of the channel on the phase slipping at low pressure and in vacuum. The dependences of the phase slipping on the physical properties of the flow and the geometric dimensions of the channel are multivalued and interrelated [3, 7]. In certain situations a decrease in the channel diameter can lead to an increase in the phase slipping, while in other situations there can be a decrease. The actual influence of the channel diameter on the pulses is governed by the kinematic properties

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TABLE 1. Experimental Characteristics

Reference	Mixture	$P\cdot 10^{-4}$ N/m^2	D, mm	$\nu_1\cdot 10^6$, m^2/sec	ρ_1, kg/m^3	$\frac{l_{exp}}{D}\left(\frac{l_{exp}}{D_{equ}}\right)$	$\frac{l_{st}}{D}\left(\frac{l_{st}}{D_{equ}}\right)$	w_{01}, m/sec	w_{02}, m/sec	S_{r0}	S_r	Arbitrary designation
Circular tubes												
[1,2,3]	W–A	10—12	32,7	0,71	985	62	50	0,15—1,25	1—30	1,95	0,91	1
[1,2,3]	W–A	6,10—8,7	32,7	0,71	985	62	50	0,15—1,25	2—9	1,95	0,96	1
[1,3]	U–A	10,5—12,7	32,7	9,25	1230	62	50	0,15—1,25	1,5—32,5	1,62	0,99	1
[1,3]	U–A	5,5—8	32,7	9,25	1230	62	50	0,15—1,25	2,5—10	1,62	1,10	1
[1,3]	U–A	10,7—12,7	32,7	28,9	1290	62	50	0,15—1,25	1,5—27,5	1,39	1,18	1
[1,3]	U–A	11,4—13,1	32,7	91,5	1330	62	50	0,15—1,25	1,5—22	1,39	1,33	1
[3]	W–A	10,6—13	12,2	0,71	985	171	50	0,15—1,10	1,19—9,5	0	1,05	1
[6,7]	W–A	9,81*	250	0,71	985	15,2	0	0,28—1,62	1,5—28	1,40	1,07	2
[6,7]	W–A	9,81*	500	0,71	985	7,6	0	0,06—0,82	0,4—9,5	1,12	1,13	2
[6,7]	L–A	9,81*	500	0,71	1220	7,6	0	0,08—0,57	1,2—9,5	1,12	1,09	2
[6,7]	U–A	9,81*	500	3,03	1120	7,6	0	0,18—0,66	1,4—8,5	1,12	1,20	2
[6,7]	U–A	9,81*	500	9,25	1235	7,6	0	0,18—0,66	1,2—8,7	1,12	1,20	2
[6,7]	U–A	9,81*	500	36,1	1290	7,6	0	0,18—0,62	1,4—9,0	1,12	1,20	2
[6,7]	U–A	9,81*	500	92,0	1330	7,6	0	0,18—0,66	1,5—8,5	1,12	1,20	2
[4,5]	W–A	9,81*	56	0,81	985	62,5	0	0,02—0,80	0,1—20	1,45	0,94	2
Annular channels												
[4,5]	W–A	10,2—11,4	23,2	0,71	985	151	0	0,02—0,6	0,1—12,5	1,64	1,06	3
[4,5]	W–A	10,2—11,5	6,1	0,71	985	575	0	0,05—0,56	0,24—18,6	1,41	1,24	3
[4,5]	U–A	9,81*	6,1	16,7	1250	575	0	0,12—0,42	0,32—9,7	1,41	1,57	3
[4,5]	L–A	9,81*	6,1	0,86	1160	575	0	0,16—0,34	0,34—2,48	1,41	1,06	3
[4,5]	G–A	9,81*	23,2	5,9	1130	151	0	0,1—0,16	0,27—5,48	1,64	1,10	3
[4,5]	U–A	9,81*	23,2	16,7	1250	151	0	0,15—0,38	0,64—4,29	1,45	1,27	3
[4,5]	L–A	9,81*	23,2	0,86	1160	151	0	0,05—0,46	0,1—1,5	1,45	1,01	3
[4,5]	W–A	5,9—6,0	23,2	0,84	986	151	0	0,02—0,6	0,2—11	1,97	1,13	3
[4,5]	W–A	5,9—6,0	6,1	0,84	985	174	0	0,1—0,5	0,5—10	1,41	1,24	3

of the flow (the reduced velocities of the phases, the mixture velocities, etc.) and by the physical properties of the flow. For example, the actual gas content in a vertical air–water flow (P = 1 bar) at w_{02} < 3.2 m/sec in a tube with D = 12.2 mm is higher than in a tube with D = 32.7 mm under otherwise equal conditions; w_{02} > 3.2 m/sec we have the opposite situation. Furthermore, while the slipping increases significantly with increasing viscosity in a tube with D = 32.7 mm, the slipping in a tube with D = 500 mm varies only slightly as the viscosity is changed by a factor of 130 [7].

The multivalued nature of, and the relations among, the effects of the various dimensional parameters on the process offer further motivation for working out relations for determining the characteristics of two-phase flows on the basis of the dimensionless criteria and their combinations.

The concept of self-similarity is important in the case of multiparameter relations [11]. We believe it is useful to introduce the concepts of "anisotropy" and "isotropy" of the self-similarity: A dimensionless parameter usually incorporates several dimensional parameters. "Isotropic" self-similarity should be understood here as self-similarity with respect to a dimensionless parameter upon a change in any of the dimensional parameters, while "anisotropic" self-similarity represents self-similarity with respect to a dimensionless parameter upon a change in only certain of the dimensional parameters. By working out complexes of such anisotropic criteria we can greatly simplify the original criterial dependence.

We have carried out an extremely extensive and careful search for self-similarity of the governed parameters φ, w_r (w_r/w_m), w_2 (w_2/w_m), and s with respect to the dimensionless and dimensional governing parameters appearing in system (1). As a result we conclude that the best quantity to adopt as the governed parameter for low-pressure and vacuum conditions is the relative velocity of the different phases w_r, more precisely, the dimensionless phase-slipping parameters S_{r0} and S_r. At low-pressures and in vacuum, according to our research [1-8], the function w_r = f (w_{02}) is approximately linear and can be written as

$$w_r = w_{r0} + S_r w_{02} \tag{2}$$

or

$$w_r = S_{r0}w + S_r w_{02}, \quad (3)$$

where

$$S_{r0} = w_{r0}/w, \quad (4a)$$

$$S_r = (w_r - w_{r0})/w_{02}, \quad (4b)$$

$$w \simeq c\sqrt{gD}. \quad (4c)$$

Relation (4c), obtained for circular tubes [12], is used in the present paper for annular channels by replacing D by the equivalent diameter $D_{equ} = D - d_i$.

Taking this discussion into account, in constructing the generalization we use the multifactor correlation relationships

$$S_{r0} = f_1(\beta,\ Re_m\ Fr_m\ We,\ \bar{\rho},\ \bar{\mu},\ k/D,\ \alpha), \quad (5)$$

$$S_r = f_2(\beta,\ Re_m\ Fr_m\ We,\ \bar{\rho},\ \bar{\mu},\ k/D,\ \alpha), \quad (6)$$

after first converting them to a form more convenient for generalizing the concrete experimental data in Table 1.

We eliminate the parameters α and k/D from (5) on the basis of the following considerations: The angle between the experimental tube and the vertical is constant in all these experiments, equal to $\alpha = 0$; the ranges of the Reynolds number of the heavy phase, Re_1, and the relative roughness k/D are such that the experimental data used in this analysis corresponds to the range of smooth tubes [9, 13]. The parameter S_{r0} corresponds to the conditions $\beta \to 0$, $w_{02} \to 0$, $w_m \to w_0$, so that we are completely justified in eliminating β from system (5) and replacing the parameters Re_m and Fr_m by Re_0 and Fr_0. Then system (5) becomes

$$S_{r0} = f_1(Re_0,\ Fr_0,\ We,\ \bar{\rho},\ \bar{\mu}). \quad (7)$$

The quantity w_0 ($0.02 \leq w_0 \leq 2$ m/sec in the experiments analyzed here) does not have any significant effect on $S_{r0}(w_{r0})$ (see, e.g., [3, 6, 7, 8]); i.e., the situation is such that the parameters Re_0 and Fr_0 "anisotropically" influence the governed parameter S_{r0}. We therefore combine Re_0 and Fr_0 into a single combination in order to eliminate w_0:

$$Re_0 \cdot Fr_0^{-1/2} = gl^3/\nu_1^2)^{1/2} = Ga^{1/2}. \quad (8)$$

Using (8) we convert system (5) to

$$S_{r0} = f_1(Ga,\ We,\ \bar{\rho},\ \bar{\mu}). \quad (9)$$

In the data analyzed here (Table 1) the change in $\bar{\mu}$ is due primarily to μ_1; i.e., the influence of the viscosity of the heavy phase on the process can be taken into account wholly by means of the criterion Ga. For the generalization we therefore use system (5) in the form

$$S_{r0} = f_1(Ga,\ We,\ \bar{\rho}). \quad (10)$$

We simplify system (6), from which we immediately eliminate α and k/D on the basis of the arguments above. Since the dimensionless parameter S_r is self-similar with respect to $w_m(\beta, w_{02}, w_{01})$, the parameters Re_m and Fr_m are combined in a manner such that we can eliminate w_m from them:

$$Re_m \cdot Fr_m^{-1/2} = (gl^3/\nu_m^2)^{1/2} = Ga_*^{1/2}. \quad (11)$$

Here Ga* is specific; in it, ν_1 is replaced by the mixture viscosity ν_m, determined from

$$1/\nu_m = (1-\beta)/\nu_1 + \beta/\nu_2. \quad (12)$$

For the experiments analyzed here (Table 1) the range of the kinematic viscosity of the lighter phase, ν_2, is two orders of magnitude smaller than the range of ν_1; furthermore, the slipping parameter S_r is self-similar with respect to β. Then we can replace ν_m in Ga* by ν_1 and obtain the familiar parameter Ga. In connection with the introduction of the parameter Ga on the basis of the considerations above, we eliminate the simplex $\bar{\mu}$ from (6). After the conversion, system (6) becomes

$$S_r = f_2(\text{Ga}, \text{We}, \bar{\rho}). \quad (13)$$

The introduction of the Galileo number among the governing parameters can be justified in a slightly different manner, by noting that the original differential equations from which system (1) was derived are "smooth" (the magnitude of the discrete formations is not taken into account) and do not reflect the internal structure of turbulent gas–liquid flows. In an attempt to take into account the microscopic structure of the turbulence of the heavy phase we assume that the influence of the viscosity on the process becomes appreciable when the scale dimension of the smallest eddy l_s (l_s is the dimension of the eddies in which the turbulent-motion energy is converted into heat by viscous forces) becomes comparable to the geometric dimension of the channel. We believe that the ratio l_s/D should be used as a measure of the influence of the heavy-phase structure of the turbulent gas–liquid flow on phase slipping.

For isotropic turbulence we have [14]

$$l_s = \nu_1^{3/4}\varepsilon^{-1/4}. \quad (14)$$

Equation (14) can also be applied to anisotropic turbulence [14]. From the von Kármán–Howarth equation we find the energy dissipation rate to be

$$\varepsilon = 65(\Delta\bar{w})^3/L. \quad (15)$$

Using (15) in (14) we find

$$l_s = (L/65)^{1/4}\cdot(\nu_1/\Delta\bar{w})^{3/4}. \quad (16)$$

The relative dimension of the smallest eddies is

$$l_s/D_* = (L/65)^{1/4}\cdot(\nu_1/\Delta\bar{w})^{3/4}D_*^{-1}. \quad (17)$$

Since S_{r0} and S_r reflect the dynamics of the change in the phase slipping with changes in the physical properties of the flow and the geometric dimensions of the channel under kinematically identical conditions, we can write, with an accuracy suitable for this generalization,

$$L^{1/4}/(\Delta\bar{w})^{3/4} \sim \text{const}; \quad (18)$$

then we find

$$l_s/D_* \sim \nu_1^m/D_*. \quad (19)$$

Kutateladze [15] recommends that the quantity $\sqrt[3]{\nu_1^2/g}$ be adopted as linear. We therefore write the ratio in (17) as

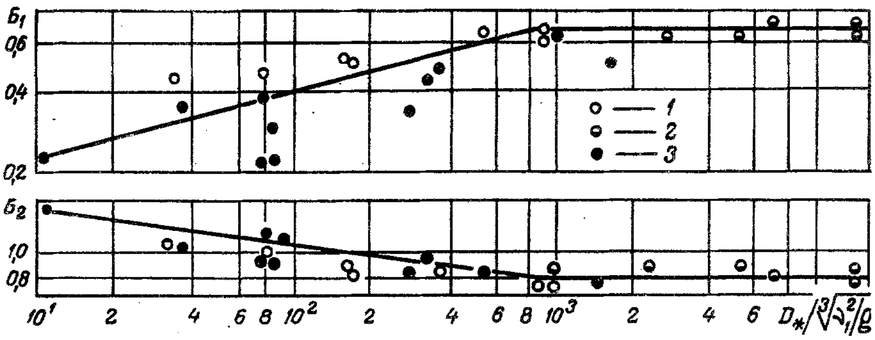

Fig. 1. Generalization of the experimental data on the actual gas content. $B_1 \equiv \frac{S_{r0}}{(1/D_*)^{0,25}\bar{\rho}^{0,05}} = f_1\left(\frac{D_*}{\sqrt[3]{\nu_1^2/g}}\right)$; $B_2 \equiv \frac{S_r(1/D_*)^{0,04}}{\bar{\rho}^{0,05}} = f_2\left(\frac{D_*}{\sqrt[3]{\nu_1^2/g}}\right)$; the arbitrary designations 1-3 correspond to those in Table 1.

$$\frac{\nu_1^m}{D_*} \sim \frac{\sqrt[3]{\nu_1^2/g}}{D_*} = \mathrm{Ga}^{-1/3}. \tag{20}$$

These manipulations reflect the physical meaning of introducing the Galileo number Ga in the system of governing parameters.

Systems (10) and (13) should be used to generalize the experimental results, with the customary parameter We replaced by the specific parameter We*:

$$S_{r0},\ S_r = f(\mathrm{Ga},\ \mathrm{We}^*,\ \bar{\rho}). \tag{21}$$

The parameter We* is modified through an account of the specific experimental data (Table 1), in which the quantity $\sqrt{\sigma/(\gamma_1-\gamma_2)}$ varies only slightly. With an accuracy suitable for this generalization we set $\sqrt{\sigma/(\gamma_1-\gamma_2)} \approx \mathrm{const}$ (for estimates we used the results of the earlier generalization in [3]) and thus

$$\mathrm{We}^* \approx \mathrm{const}/D_* \sim 1/D_*. \tag{22}$$

The experimental data are shown in Fig. 1 in the coordinates

$$S_{r0}(1/D_*)^{-0,25}\bar{\rho}^{-0,05} = f_1(D_*/\sqrt[3]{\nu_1^2/g}), \tag{23}$$

$$S_r(1/D_*)^{0,04}\bar{\rho}^{-0,05} = f_2(D_*/\sqrt[3]{\nu_1^2/g}). \tag{24}$$

The exponent of the simplex $\bar{\rho}$ is adopted in accordance with the generalization in [3].

In the region $\mathrm{Ga}^{1/3} > 800$ the parameters S_{r0} and S_r are self-similar with respect to Ga; i.e., the heavy-phase viscosity does not significantly affect the phase slipping over a broad range of channel diameters in this region. At $\mathrm{Ga}^{1/3} < 800$ the scatter of points with respect to the upper generalizing curve is very large. This is a reasonable result: As the velocity w_{r0} approaches zero, the relative error in its determination increases rapidly. The error which the quantity $S_{r0}(w_{r0})$ causes in the calculation of the actual gas content $\bar{\varphi}$ is appreciable only at low gas flow rates, since at intermediate and large flow rates the phase slipping is governed primarily by the parameter S_r.

The experimental results shown in Fig. 1 are described analytically by

$$S_{r0}=0.123(\bar{\rho})^{0.05}(1/D_*)^{0.25}(D_*/\sqrt[3]{\nu_1^2/g})^{0.25}, \quad (25)$$

$$S_r = 1.85(\bar{\rho})^{0.05}(1/D_*)^{-0.04}(D_*/\sqrt[3]{\nu_1^2/g})^{-0.125} \quad (26)$$

for $Ga^{1/3} < 800$;

$$S_{r0} = 0{,}65\,(1/D_*)^{0{,}25}\bar{\rho}^{0{,}05}, \quad (27)$$

$$S_r = 0{,}80\,(1/D_*)^{-0{,}04}\bar{\rho}^{0{,}05} \quad (28)$$

for $Ga^{1/3} > 800$.

Accordingly, to determine the actual gas content in the motion of low-viscosity and high-viscosity flows in tubes and annular channels with equivalent diameters D_{equ} = 6-1000 mm we can use

$$\varphi = 0{,}5\left(\frac{w_c}{S_{r0}w + S_r w_{02}} + 1\right) - \left[0{,}25\left(\frac{w_c}{S_{r0}w + S_r w_{02}} + 1\right)^2 - \frac{w_{02}}{S_{r0}w + S_r w_{02}}\right]^{1/2}, \quad (29)$$

where S_{r0} and S_r are determined for (25)-(28).

NOTATION

φ, β, actual and expended volume gas contents; D, geometric diameter; d_i, outside diameter of the inner wall in an annular channel; D_{equ}, hydraulic (equivalent) diameter; l, linear dimension; l_{exp}, length of experimental region; l_{st}, length of stabilization region; $\mu = \mu_1/\mu_2$, relative dynamic viscosity; μ_1, μ_2, dynamic viscosity of the heavy and light phases, respectively; $\bar{\rho} = \rho_1/\rho_2$, relative density; ρ_1, ρ_2, densities of the heavy and light phases, respectively; $We = \sigma/[\gamma_1-\gamma_2)D_*^2]$, Weber number; σ, surface tension; γ_1, γ_2, specific gravities of the heavy and light phases, respectively; $Fr_m = w_m/(g/D_*)$, Froude number of the mixture w_m, mixture velocity; g, acceleration due to gravity; $Re_m = w_m/(gD_*)$, Reynolds number of mixture; ν_m, kinematic viscosity of mixture; P, pressure; w_{01}, w_{02}, reduced velocities of the heavy and light phases; w_r, relative velocity of the different phases; w_1, w_2, actual velocities of the heavy and light phases; $S = w_2/w_1$, phase-slipping parameter; w_0, circulation velocity; S_{r0}, S_r, dimensionless phase-slipping parameters; w, velocity of the relative motion of a single large bubble or "projectile; Re_0, Fr_0, Reynolds and Froude numbers or single-phase flow; w_{r0}, relative velocity of the different phases for the case in which w_{02} tends toward zero; Ga, galileo number; α, angle between the tube and the vertical; ν_1, ν_2, kinematic viscosities of the heavy and light phases; ε, rate of energy dissipation; L, scale length of the turbulence; $\Delta\bar{w}$, pulsation velocity; D_*, linear dimension (for a tube, $D_* = D$; for an annular channel, $D_* = D_{equ}/2$); c, coefficient, c = 0.35 [12]; W–A, water–air mixture; U–A mixture of a sugar solution and air; L–A, mixture of a salt solution and air; G–A, mixture of a glycerin solution and air.

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